

Exercise 10

Use $i = (0, 1)$ and $y = (y, 0)$ to verify that $-(iy) = (-i)y$. Thus show that the additive inverse of a complex number $z = x + iy$ can be written $-z = -x - iy$ without ambiguity.

Solution

Evaluate the left side.

$$\begin{aligned} -(iy) &= -[(0, 1)(y, 0)] \\ &= -[(0 - 0, y + 0)] \\ &= -(0, y) \\ &= (-1, 0)(0, y) \\ &= (0 - 0, 0 - y) \\ &= (0, -y) \end{aligned}$$

Evaluate the right side.

$$\begin{aligned} (-i)y &= (0, -1)(y, 0) \\ &= (0 - 0, -y + 0) \\ &= (0, -y) \end{aligned}$$

Consequently, $-(iy) = (-i)y$ is verified.